The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

**PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

### 1. REPORT DATE (DD-MM-YYYY)
February 11, 2005

### 2. REPORT TYPE
Final

### 3. DATES COVERED (From - To)
1 July 2001 - 30 September 2004

### 4. TITLE AND SUBTITLE
Imaging, Time-reversal and Communications in a Random Medium

### 5. a. CONTRACT NUMBER

### 5b. GRANT NUMBER
F49620-01-1-0465

### 5c. PROGRAM ELEMENT NUMBER

### 5d. PROJECT NUMBER

### 5e. TASK NUMBER

### 5f. WORK UNIT NUMBER

### 6. AUTHOR(S)
George C. Papanicolaou

### 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Department of Mathematics
Stanford University
Stanford, CA 94305

### 8. PERFORMING ORGANIZATION REPORT NUMBER

### 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
4015 Wilson Blvd
Mail Room 713
Arlington, VA 22203

### 10. SPONSOR/MONITOR'S ACRONYM(S)
AFOSR

### 11. SPONSOR/MONITOR'S REPORT NUMBER(S)

### 12. DISTRIBUTION/AVAILABILITY STATEMENT
Distribution Statement A. Approved for public release; distribution is unlimited.

### 13. SUPPLEMENTARY NOTES

### 14. ABSTRACT
The core subject of my research supported by AFOSR is imaging in random media applications in ultrasound imaging as well as radar remote sensing and the use of time-reversal in array-to-array communications. We have made considerable progress in both imaging and time reversal during the three year period. In addition to Borcea, the other researchers attached to this project were: L. Borcea, J. Berryman, C. Tsogka, A Kim and P. Blomgren, D. Berebichez, C. Deravez.

### 15. SUBJECT TERMS

### 16. SECURITY CLASSIFICATION OF:
a. REPORT
U

b. ABSTRACT
U

c. THIS PAGE
U

### 17. LIMITATION OF ABSTRACT
UU

### 18. NUMBER OF PAGES
UU

### 19. a. NAME OF RESPONSIBLE PERSON
George C. Papanicolaou

b. TELEPHONE NUMBER (Include area code)
650-723-2081

---

Standard Form 298 (Rev. 8/98)
Prepared by ANSI Std. 239.18
Final Report for Research on "Imaging, Time-reversal and Communications in a Random Medium"
under grant AFOSR F49620-01-1-0465
for the period July 1, 2001 to September 30, 2004

George C. Papanicolaou
Department of Mathematics, Stanford University
Stanford CA 94305; phone 650 7232081, fax 650 7254066
email papanico@math.stanford.edu; URL http://georgep.stanford.edu

February 11, 2005

1 Introduction

The core subject of my research supported by AFOSR is imaging in random media with applications in ultrasound imaging as well as radar remote sensing and the use of time-reversal in array-to-array communications.

We have made considerable progress in both imaging and time reversal during the three year period. In addition to Borcea, the other researchers attached to this project were: L. Borcea, J. Berryman, C. Tsogka, A. Kim and P. Blomgren, D. Berebichez, G. Derveaux.

Here are our main results

- Introduction and analysis of interferometric array imaging for cluttered media. Papers 1,2,3,9.


- Broadband array imaging of small scatterers in clutter using the singular value decomposition. Papers 5,6.

- Study of super-resolution and statistical stability in Time Reversal, including numerical simulations. Papers 7,8.

- Time reversal communication systems. Analysis of cross-channel interference (spatial focusing) and intersymbol interference (time focusing). Role of time-reversal in assessing these issues. Papers 13-17.

- Various papers (12, 18, 20) in wave propagation and imaging related to this work mathematically.
2 Relevant publications of G. Papanicolaou and collaborators in 2002-2005

All papers cited here that have not yet appeared in print can be accessed from http://georgep.stanford.edu (in compressed PostScript or pdf format)

**IMAGING and TIME-REVERSAL**

1. Interferometric array imaging in clutter, Liliana Borcea, George Papanicolaou and Chrysoulu Tsogka. Submitted to Inverse Problems.
   
   Abstract:
   
   We introduce a space-time interferometric array imaging functional that provides statistically stable images in cluttered environments. We also present a resolution theory for this imaging functional that relates the space-time coherence of the data to the range and cross-range resolution of the image. Extensive numerical simulations illustrate the theory and address some implementation issues.

   
   Abstract:
   
   In time reversal, an array of transducers receives the signal emitted by a localized source, time reverses it and re-emits it into the medium. The emitted waves back-propagate to the source and tend to focus near it. In a homogeneous medium, the cross-range resolution of the refocused field at the source location is $\lambda_0 L/a$, where $\lambda_0$ is the carrier wavelength, $L$ is the range and $a$ is the array aperture. In a noisy (random) medium, the cross-range resolution is improved beyond the homogeneous diffraction limit because the array can capture waves that move away from it at the source, but get scattered onto it by the inhomogeneities. We refer to this phenomenon as super-resolution of the time reversal process in random media. Super-resolution implies in particular that, because of multipathing, the array appears to have an effective aperture that is greater than $a$. We distinguish two types of effective apertures: (i) the narrow-band one $a_\varepsilon$ that depends on the scattering medium and (ii) the broad-band one $A_\varepsilon$ which is determined by $a_\varepsilon$ and by the bandwidth of the signal. In remote sensing regimes ($L \gg a$), the influence of the aperture $a$ of the array on the focusing resolution is negligible and focusing is essentially determined by $a_\varepsilon$. However, at a moderate source range the array aperture does play a role and focusing depends on $A_\varepsilon$. Since $a_\varepsilon$ depends on the scattering medium, it is not known. In this paper we present a robust procedure for estimating $a_\varepsilon$ from the signals received at the array. The interest of this estimation is twofold: First, knowing $a_\varepsilon$ permits assessing quantitatively super-resolution in time reversal. Thus, it provides an estimation of the refocusing resolution that can be achieved by the time reversal process in cluttered media for applications such as spatially localized, secure communications. Second, as we show in this paper, $a_\varepsilon$ quantifies in an explicit way the loss of resolution in array imaging.


   
   Abstract:
   
   When a signal is emitted from a source, recorded by an array of transducers, time reversed and re-emitted into the medium, it will refocus approximately on the source location. We analyze the
refocusing resolution in a high frequency, remote sensing regime, and show that, because of multiple scattering, in an inhomogeneous or random medium it can improve beyond the diffraction limit. We also show that the back-propagated signal from a spatially localized narrow-band source is self-averaging, or statistically stable, and relate this to the self-averaging properties of functionals of the Wigner distribution in phase space. Time reversal from spatially distributed sources is self-averaging only for broad-band signals. The array of transducers operates in a remote-sensing regime so we analyze time reversal with the parabolic or paraxial wave equation.


Abstract:

We present a general method for estimating the location of small, well-separated scatterers in a randomly inhomogeneous environment using an active sensor array. The main features of this method are (i) an arrival time analysis of the echo received from the scatterers, (ii) a singular value decomposition of the array response matrix in the frequency domain, and (iii) the construction of an objective function in the time domain that is statistically stable and peaks on the scatterers. By statistically stable we mean here that the objective function is self-averaging over individual realizations of the medium. This is a new approach to array imaging that is motivated by time reversal in random media, analyzed in detail previously. It combines features from seismic imaging like arrival time analysis with frequency-domain signal subspace methodology like MULTiple Signal Classification (MUSIC). We illustrate the theory with numerical simulations for ultrasound.


Abstract:

Analysis of array data from acoustic scattering in a random medium with a small number of isolated targets is performed in order to image and, thereby, localize the spatial position of each target. Because the host medium has random fluctuations in wave speed, the background medium is itself a source of scattered energy. We assume, however, that the targets are sufficiently larger and/or more reflective than the background fluctuations so that a clear distinction can be made between targets and background scatterers. In our numerical simulations we use nonreflective boundary conditions so as to isolate the effects of the host randomness from those of the spatial boundaries, which can then be treated in a separate analysis. We show that the key to successful imaging is finding statistically stable functionals of the data whose extreme values provide estimates of scatterer locations. The best ones are related to the eigenfunctions and eigenvalues of the array response matrix, just as one might expect from prior work on array data processing in complex scattering media having homogeneous backgrounds. The specific imaging functionals studied include matched-field processing and linear subspace methods, such as MUSIC (MULTiple Signal Classification). But statistical stability is not characteristic of the frequency domain, which is often the province of these methods. By transforming back into the time domain after first diagonalizing the array data in the frequency domain, we can take advantage of both the time-domain stability and the frequency-domain orthogonality of the relevant eigenfunctions.


Abstract:

We analyze theoretically and with numerical simulations the phenomenon of super-resolution in time-reversal acoustics. A signal that is recorded and then re-transmitted by an array of transducers, propagates back though the medium and refocuses approximately on the source that emitted it. In a homogeneous medium, the refocusing resolution of the time-reversed signal is limited by
diffraction. When the medium has random inhomogeneities the resolution of the refocused signal can in some circumstances beat the diffraction limit. This is super-resolution. We give a theoretical treatment of this phenomenon and present numerical simulations which confirm the theory.


Abstract:
We present numerical computations that reproduce the time reversal experiments of Draeger, Cassereau and Fink, where ultrasound elastic waves are time reversed back to their source with a Time Reversal Mirror (TRM) in a fluid adjacent to the solid. We also show numerically that multipathing caused by random inhomogeneities improves the focusing of the back-propagated elastic waves beyond the diffraction limit seen previously in acoustic wave propagation, which is called super-resolution. A theoretical explanation of the robustness of super-resolution along with several numerical computations that support this explanation is in our previous studies. Time reversal with super-resolution can be used in non-destructive testing and, in a different way, in imaging with active arrays.


TIME-REVERSOAL COMMUNICATIONS


Abstract:
We study a multiple-input/single-output underwater communication system that applies time reversal to transmit signals so that they focus spatially and compress temporally on the intended receiver. Our simulations model an underwater acoustic channel as a waveguide. The results verify the theory that spatial focusing depends strongly on the product of the delay spread and the bandwidth. Because of spatial focusing at the intended receiver, this system has a low probability of intercept. Even though signals are compressed temporally, there remains intersymbol interference, especially when the delay spread-bandwidth product is not too large. However, it is possible to remove nearly all intersymbol interference by pre-equalizing the channel. We show that the introduction of zero-forcing pre-equalization does not alter significantly the spatial focusing properties of time-reversal.


Abstract:
Using broadband radio wireless measurements in an indoor environment we demonstrate the remarkable space-time focusing properties of signal transmission with time reversal.

15. Characterization of space-time focusing in time reversed random fields, C.

Abstract:
This paper proposes various metrics to characterize space-time focalization resulting from application of time reversal techniques in richly scattering media. The concept and goals of time reversal are presented. Then, pertinent metrics describing both the time and space focalization effects are outlined. Finally, two examples, based on a model of discrete and continuous scattering media, are used to illustrate how the proposed metrics vary as a function of various system and channel parameters, such as the bandwidth, delay and angle spreads, number of antennas, etc.


Abstract:
Fixed wireless access channels in the 5GHz band have been measured with 8 element uniform linear antenna arrays at both ends, thus providing an 8x8 configuration. The measurements were performed at 3 different locations in downtown Toronto. The application of the time reversal (TR) technique in a multiple input-single output (MISO) can reduce the delay spread of the channel impulse response by a factor of 2-3, depending on the power allocation scheme.


OTHER RELATED WORK

